Assignment # 7	Due on December 18th in class

1. Prove that $B(\mathbb{C}^n, \mathbb{C})$ is isomorphic (as a normed space) to $\mathbb{C}^n$. To be more precise, you need to show that there exists a map

$$\rho\colon B(\mathbb{C}^n, \mathbb{C}) \longrightarrow \mathbb{C}^n,$$

such that

i) ρ is bijective (both injective and surjective)

ii) ρ is linear. In other words, for all $x, y \in \mathbb{C}^n$ and $\lambda \in \mathbb{C}$, we have

$$\rho(x+y) = \rho(x) + \rho(y) \text{ and } \rho(\lambda x) = \lambda \rho(x).$$

iii) ρ preserves the norm structure. In other words, for all $x \in \mathbb{C}^n$, we have $\|\rho(x)\| = \|x\|$.

2. Assuming we have the following fact (which can be derived from the Hahn-Banach Theorem):

" Let X be a Banach space. Then for any $x \in X$ with $x \neq 0$, there exists a bounded linear functional on f on X (in other words, $f \in B(X, \mathbb{C})$) such that $f(x) \neq 0$. "

Consider the following map

$$\rho\colon X \longrightarrow X^{**},\ x \mapsto \rho(x) \text{ with } \rho(x)(f) = f(x)\ \forall\, f \in X^*.$$

Prove that the map ρ is linear and injective.